An Analytical Design-Optimization Method for Electric Propulsion Systems of Multicopter UAVs With Desired Hovering Endurance

Xunhua Dai, Quan Quan, Jinrui Ren, and Kai-Yuan Cai

***Abstract*—Multicopters are becoming increasingly important both in civil and military fields. Currently, most multicopter propulsion systems are designed by experience and trial-and-error experiments, which are costly and ineffective. This paper proposes a simple and practical method to help designers find the optimal propulsion system according to the given design requirements. First, the modeling methods for four basic components of the propulsion system including propellers, motors, electric speed controls, and batteries are studied, respectively. Second, the whole optimization design problem is simplified and decoupled into several subproblems. By solving these subproblems, the optimal parameters of each component can be obtained, respectively. Finally, based on the obtained optimal component parameters, the optimal product of each component can be quickly located and determined from the corresponding database. Experiments and statistical analyses demonstrate the effectiveness of the proposed method. The proposed method is so fast and practical that it has been successfully applied to a web server to provide online optimization design service for users.**

***Index Terms*—Design optimization, multicopter, propulsion system, unmanned aerial vehicle (UAV).**

I. Introduction

DURING recent years, multicopter unmanned aerial vehicles (UAVs) have become increasingly popular both in civil and military fields [1], [2], along with other fields such as aerial photography, plant protection, and package delivery. Limited by the present battery technology, the flight time (hovering endurance) of multicopters is still too short for most applications. Because the performance and efficiency of a multicopter directly depend on the propulsion system, the design optimization for multicopter propulsion systems is urgently needed to increase the flight time.

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The design-optimization problem studied in this paper is to find the optimal combination of propulsion-system components to satisfy the given hovering-endurance requirement, and the obtained propulsion system should have smaller weight and higher efficiency as possible. A typical multicopter propulsion system usually consists of four basic components: 1) propeller; 2) BrushLess Direct-Current (BLDC) motor; 3) electronic speed control (ESC); and 4) lithium polymer (LiPo) battery [3]. Traditional methods to determine a propulsion system are usually based on experience and trial-and-error experiments. Considering that there are thousands of component products on the market, it is a costly and time-consuming work for the traditional design methods. Meanwhile, in the whole design process of a multicopter system, the propulsion system needs to be repeatedly modified according to the actual controlled system until all the performance requirements and safety requirements are met. According to [4] and [5], a more efficient way is to simultaneously design the body system (including the propulsion system) and the control system, subject to the optimal objective and additional constraints. Therefore, automatic technologies for the design and optimization of propulsion systems will be beneficial for reducing the prototyping needs for the whole multicopter system, thereby minimizing the costs of development and manufacturing. For such reasons, this paper proposes a simple, practical, and automatic design-optimization method to help designers quickly find the optimal propulsion system according to the given design requirements.

In our previous work [6], on the basis of the mathematical modeling methods for the components of propulsion systems, a practical method was proposed to estimate the flight performance of multicopters according to the given propulsion-system parameters. In fact, the study in this paper is the reverse process of our previous work, namely estimating the optimal propulsion-system parameters according to the given design requirements. This problem is more complicated and difficult because the number of design requirements is much less than the number (more than 15) of the propulsion-system parameters.

There are many studies on the mathematical modeling [7]–[9], the efficiency analysis [10], [11], and the performance estimation [6], [12] of multicopter propulsion systems. To our best knowledge, there are a few studies on the design optimization of multicopter propulsion systems. Most of them adopt numerical methods (fixed-wing aircraft [13] and multicopters [12]) to search and traverse all the possible propulsion-system combinations in the database based on the proposed cost functions. The multicopter-optimization problem is described as a mixed integer linear program, and solved with the Cplex optimizer in [14] and [15]. However, these numerical methods have the following problems.

1) A large and well-covered product database is required for a better optimization effect.
2) The calculation speed is slow when there are large numbers of products in the database because the amount of product combinations is huge [The algorithm complexity is $O(n^4)$, where n is the number of database products].

In order to solve the aforementioned problems, this paper proposes an analytical method to estimate the optimal parameters of the propulsion-system components. First, the modeling methods for each component of the propulsion system are studied, respectively, to describe the problem with mathematical expressions. Second, the whole problem is simplified and decoupled into several small problems. By solving these subproblems, the optimal parameters of each component can be obtained, respectively. Finally, based on the obtained parameters, selection algorithms are proposed to determine the optimal combination of the propeller, the motor, the ESC, and the battery products from their corresponding databases.

The contributions of this paper are as follows.

1) An analytical method to solve the design-optimization problem of multicopter propulsion systems is proposed for the first time.
2) The conclusion obtained by the theoretical analysis has a guiding significance for the multicopter design.
3) Compared to the numerical traversal methods, the proposed method reduces the algorithm complexity from $O(n^4)$ to $O(n)$, which is faster and more efficient for practical applications.
4) The optimization algorithm proposed in this paper has been published as a subfunction in an online toolbox (URL: www.flyeval.com/recalc.html) to estimate the optimal propulsion system with the given design requirements.

This paper is organized as follows. Section II gives a comprehensive analysis of the design-optimization problem to divide it into 12 subproblems is provided. In Section III, the modeling methods for each component of the propulsion system are studied to describe the subproblems with mathematical expressions. In Section IV, the subproblems are solved, respectively, to obtain the optimal components of the desired propulsion system. In Section V, statistical analyses and experiments are performed to verify the proposed method. In Section VI, conclusion is presented.

II. Problem Formulation

A. Design Requirements of Propulsion Systems

The role of a propulsion system is to continuously generate the desired thrust within the desired time of endurance for a multicopter. The design requirements for a propulsion system are usually described by the following parameters:

1) the number of propulsion units, $n_{\rm p}$;
2) the hovering thrust of a single propeller, $T_{\rm hover}$ (unit: N), under hovering mode in which the multicopter stays fixed in the air;
3) the maximum thrust of a single propeller, $T_{\rm max}$ (unit: N), under full-throttle mode in which the autopilot gives the maximum throttle signal;
4) the nominal flight altitude, $h_{\rm hover}$ (unit: m);
5) the flight time, $t_{\rm hover}$ (unit: min), under the hovering mode.

This paper focuses only on studying the design optimization of propulsion systems with an assumption that $T_{\rm hover}$ and $T_{\rm max}$ are known parameters. Although the propulsion-system parameters $T_{\rm hover}$ and $T_{\rm max}$ are usually not directly available, according to our previous research in [6], yet $T_{\rm hover}$ and $T_{\rm max}$ can be obtained by giving aerodynamic coefficients, airframe parameters, and the kinematic performance requirements. For example, for common multicopters, the hovering thrust $T_{\rm hover}$ can be obtained by the total weight of the multicopter, $G_{\rm total}$ (unit: N), as:

$$T_{\rm hover} = \frac{G_{\rm total}}{n_{\rm p}}. \tag{1}$$

The kinematic performance of a multicopter is directly determined by the thrust ratio $\gamma \in (0, 1)$ which is defined as

$$\gamma \triangleq \frac{T_{\rm hover}}{T_{\rm max}} \tag{2}$$

where thrust ratio γ describes the remaining thrust for the acceleration movement, which further determines the maximum forward speed and the wind resistance ability of a multicopter. Therefore, designers should estimate the desired γ (usually $\gamma = 0.5$ is selected for common multicopters) according to the kinematic performance requirements of the multicopter. Then, the desired full-throttle thrust $T_{\rm max}$ of the propulsion system can be obtained using (1) and (2).

TABLE I
Propulsion-System Parameters

Items	Parameters
Propeller	$\Theta_{\rm p} \triangleq$ {Diameter $D_{\rm p}$ (m), Pitch Angle $\varphi_{\rm p}$ (rad), Blade Number $B_{\rm p}$}
Motor	$\Theta_{\rm m} \triangleq$ {Nominal Maximum Voltage $U_{\rm mMax}$ (V), Nominal Maximum Current $I_{\rm mMax}$ (A), KV Value $K_{\rm V}$ (RPM/V), No-load Current $I_{\rm m0}$ (A), Resistance $R_{\rm m}$ (Ω)}
ESC	$\Theta_{\rm e} \triangleq$ {Nominal Maximum Voltage $U_{\rm eMax}$ (V), Nominal Maximum Current $I_{\rm eMax}$ (A), Resistance $R_{\rm e}$ (Ω)};
Battery	$\Theta_{\rm b} \triangleq$ {Nominal Voltage $U_{\rm b}$ (V), Maximum Discharge Rate $K_{\rm b}$ (A), Capacity $C_{\rm b}$ (mAh), Resistance $R_{\rm b}$ (Ω)};

B. Component Parameters

The ultimate goal of the design-optimization problem is to select the optimal products from four component databases with component parameters listed in Table I, where $\Theta_{\rm p}$, $\Theta_{\rm m}$, $\Theta_{\rm e}$, and $\Theta_{\rm b}$ represent the parameter sets for propellers, motors, ESCs, and batteries, respectively. In order to ensure commonality of the method, all component parameters in Table I are the basic parameters that can be easily found in the product description pages. The detailed introduction of each parameter in Table I can also be found in [3, pp. 31–46].

The propeller pitch angle, φ_{p} (unit: rad), in Table I is defined according to the propeller diameter, D_{p} (unit: m), and the propeller pitch, H_{p} (unit: m), as:

$$\varphi_{\mathrm{p}} \triangleq \arctan \frac{H_{\mathrm{p}}}{\pi D_{\mathrm{p}}} \tag{3}$$

where H_{p} and D_{p} are usually contained in the propeller model name.

The most commonly used unit for battery voltage U_{b}, as well as motor voltage U_{mMax} and the ESC voltage U_{eMax}, is "S," which denotes the number of battery cells in series. For LiPo batteries, the voltage changes from 4.2 to 3.7 V as the battery capacity decreases from full to empty, and the average voltage 4.0 V is adopted for unit conversion. For example, for a 12S LiPo battery, $U_{\mathrm{b}} = 48$ V.

C. Optimization Constraints

1) Requirement Constraints: According to [6], t_{hover} and T_{max} can be estimated by parameters $n_{\mathrm{p}}, T_{\mathrm{hover}}, \Theta_{\mathrm{p}}, \Theta_{\mathrm{m}}, \Theta_{\mathrm{e}}$, and$\Theta_{\mathrm{b}}$. Therefore, two equality constraints can be obtained according to the design requirements in Section II-A.

2) Safety Constraints: The electric components should work within their allowed operating conditions to prevent from being burnt out. Therefore, a series of inequality constraints can be obtained for electric components including the battery, the ESC, and the motor of the propulsion system. The detailed inequality expressions will be introduced in later sections.

3) Product Statistical Constraints: In order to make sure that the obtained solution is meaningful and practical, the statistical features of the product should be considered. Otherwise, it is very possible that there is no product in reality matching with the obtained component parameters. The product features can be described by equality constraints based on the statistical models of the products in the database. In practice, products with different material, operating principle, and processing technology may have different product features; for example, LiPo batteries and Ni-MH batteries. Therefore, different statistical models should be obtained for different types of products, and designers should select the required product type to the following optimization process to improve the precision and practicability of the obtained result.

D. Optimization Problem

In practice, there are the following two methods to increase the flight time of a multicopter:

1) decrease the total weight of the multicopter to allocate more free weight for the battery capacity;
2) increase the efficiency of the multicopter propulsion system to decrease the required battery capacity.

For a typical multicopter, according to the weight statistical model in [12], the weight of the propulsionsystem is the main source (usually more than 70%) of the multicopter's weight, and the weight of the battery is further the main source (usually more than 60%) of the weight of the propulsion system. In fact, for a propulsion system, the maximum efficiency usually means the minimum battery weight (capacity). Therefore, the aforementioned two methods essentially share the same optimization objective, namely, minimizing the weight of the propulsion system.

Assuming that the total weight of the propulsion system is defined as G_{sys} (unit: N), the optimization objective of the design optimization can be described as:

$$\min_{\Theta_{\mathrm{p}},\Theta_{\mathrm{m}},\Theta_{\mathrm{e}},\Theta_{\mathrm{b}}} G_{\mathrm{sys}}. \tag{4}$$

Along with the constraints in Section II-C, the optimal solutions for the parameters of each component can be obtained using (4).

E. Problem Decomposition

According to [6], the detailed mathematical expressions for (4) are very complex because there are 15 parameters listed in Table I, which need to be optimized by solving complex nonlinear equations. As a result, decomposition and simplification are required to solve this problem.

1) Weight Decomposition: The total weight of propulsion system G_{sys} is determined by the weight of each component as:

$$G_{\mathrm{sys}} = n_{\mathrm{p}} \left(G_{\mathrm{p}} + G_{\mathrm{m}} + G_{\mathrm{e}}\right) + G_{\mathrm{b}} \tag{5}$$

where $G_{\mathrm{p}}, G_{\mathrm{m}}, G_{\mathrm{e}}$, and G_{b} (unit: N) denote the weight of the propeller, the motor, the ESC, and the battery, respectively. Therefore, on the basis of the idea of a greedy algorithm, the optimization objective of minimizing G_{sys} can be decomposed into four subproblems of minimizing the weight of each component as:

$$\min G_{\mathrm{sys}} \Rightarrow \min G_{\mathrm{p}}, \min G_{\mathrm{m}}, \min G_{\mathrm{e}}, \min G_{\mathrm{b}}. \tag{6}$$

As mentioned above, the battery weight G_{b} is the most important factor for G_{sys}, and G_{b} directly depends on battery capacity C_{b}. Since, according to the analysis in [3], battery capacity C_{b} depends on the efficiency of each component, the optimization objective of minimizing G_{b} can also be decomposed into four subproblems of maximizing the efficiency of each component as:

$$\min G_{\mathrm{b}} \Rightarrow \max \eta_{\mathrm{p}}, \max \eta_{\mathrm{m}}, \max \eta_{\mathrm{e}}, \max \eta_{\mathrm{b}} \tag{7}$$

where $\eta_{\mathrm{p}}, \eta_{\mathrm{m}}, \eta_{\mathrm{e}}$, and η_{b} denote the efficiency of the propeller, the motor, the ESC, and the battery, respectively.

Thus, by solving the eight subproblems in (6) and (7), the optimal solutions for the parameters in Table I can be obtained with the results presented in Fig. 1, where the obtained optimal solutions are marked with a subscript "Opt." Then, the optimal parameter sets for the propeller, the motor, the ESC, and the battery can be determined and represented by Θ_{pOpt}, Θ_{mOpt}, Θ_{eOpt}, and Θ_{bOpt}, respectively.

2) Product Selection Decomposition: The ultimate goal of the design optimization of the propulsion system is to determine the optimal propeller, motor, ESC, and propeller products from their corresponding databases according to the obtained optimal parameter sets $\Theta_{\mathrm{pOpt}}, \Theta_{\mathrm{mOpt}}, \Theta_{\mathrm{eOpt}}$, and Θ_{bOpt}. This problem can also be divided into four subproblems. By solving the four subproblems, the parameter sets of the obtained products are represented by $\Theta^{*}_{\mathrm{pOpt}}$, $\Theta^{*}_{\mathrm{pOpt}}$, $\Theta^{*}_{\mathrm{pOpt}}$, and $\Theta^{*}_{\mathrm{pOpt}}$.

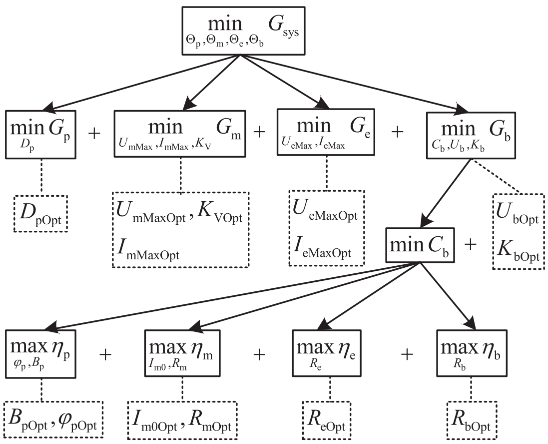

Fig. 1. Optimization objective decomposition diagram.

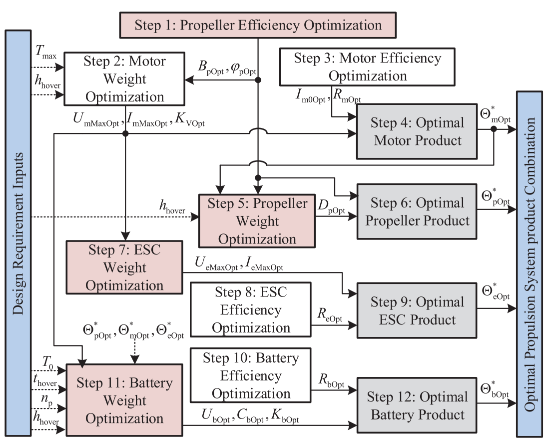

Fig. 2. Solving procedures of the propulsion-system optimization problem.

F. Solving Procedures

Using the aforementioned decomposition procedures, the whole design-optimization problem can finally be simplified, decoupled, and divided into 12 subproblems. However, because there are argument-dependent relations among the subproblems, the solving sequence should be well arranged. For example, the propeller parameters B_p, φ_p are required for motor optimization, and optimal propeller diameter, D_p, depends on the obtained motor parameters. As a result, the propeller and the motor should be treated as a whole during the solving procedures.

In this paper, the 12 subproblems will be solved separately in 12 steps, and the solving sequence is shown in Fig. 2. Moreover, Fig. 2 presents the inputs, outputs, and parameter dependence relations of each subproblem. The detailed solving methods of each step will be introduced in Section IV with 12 subsections.

III. Propulsion-System Modeling

The whole propulsion system can be modeled by the equivalent circuit [6] as shown in Fig. 3, with which the optimization subproblems can be described using mathematical expressions.

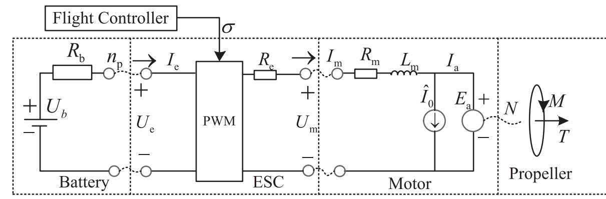

Fig. 3. Equivalent circuit model of the whole propulsion system.

A. Propeller Modeling

1) Propeller Aerodynamic Model: According to [16], the thrust force T (unit: N) and torque M (unit: N·m) of fixed-pitch propellers can be obtained using the following:

$$\begin{cases} T = C_{\mathrm{T}}\rho\left(\frac{N}{60}\right)^2 D_{\mathrm{p}}^4 \\ M = C_{\mathrm{M}}\rho\left(\frac{N}{60}\right)^2 D_{\mathrm{p}}^5 \end{cases} \tag{8}$$

where N (unit: RPM) is the propeller revolving speed, ρ is the local air density (unit: kg/m^3), C_{T} is the propeller thrust coefficient, C_{M} is the propeller torque coefficient, and D_{p} (unit: m, from Θ_{p}) is the propeller diameter.

The air density, ρ, is determined by both the local temperature T_{t} (unit: °C) and the air pressure, which is further determined by altitude h_{hover} (unit: m). According to the international standard atmosphere model [17], we have the following:

$$\rho = f_\rho\left(h_{\mathrm{hover}}\right) \triangleq \frac{273}{(273+T_{\mathrm{t}})}\left(1-0.0065\frac{h_{\mathrm{hover}}}{273+T_{\mathrm{t}}}\right)^{5.2561}\rho_0 \tag{9}$$

where the standard air density $\rho_0 = 1.293$ kg/m^3 (°C, 273 K).

The propeller coefficients C_{T} and C_{M} can be modeled using the blade element theory as presented in [6] and [9]. According to our previous work [6], a corrected model based on the propeller experimental data in [18] is given as:

$$\begin{cases} C_{\mathrm{T}} &= f_{C_{\mathrm{T}}}\left(B_{\mathrm{p}}, D_{\mathrm{p}}, \varphi_{\mathrm{p}}\right) &\triangleq k_{\mathrm{t0}} B_{\mathrm{p}}^{\alpha_{\mathrm{t}}}\varphi_{\mathrm{p}} \\ C_{\mathrm{M}} &= f_{C_{\mathrm{M}}}\left(B_{\mathrm{p}}, D_{\mathrm{p}}, \varphi_{\mathrm{p}}\right) &\triangleq k_{\mathrm{m0}} B_{\mathrm{p}}^{\alpha_{\mathrm{m}}}\left(k_{\mathrm{m1}} + k_{\mathrm{m2}}\varphi_{\mathrm{p}}^2\right) \end{cases} \tag{10}$$

where $k_{\mathrm{t0}}, k_{\mathrm{m0}}, k_{\mathrm{m1}},$ and k_{m2}, which can be obtained from [6], are constant parameters determined by the shapes and aerodynamic characteristics of the propeller blades, and $\alpha_{\mathrm{t}} < \alpha_{\mathrm{m}} \approx 1$ are correction coefficients for the interference between blades [9] on the basis of the experimental data in [18]. In practice, α_{t} is smaller than α_{m} because, according to the blade element theory [9], C_{T} suffers more blade interference than those suffered by C_{M}. Note that $k_{\mathrm{t0}}, k_{\mathrm{m0}}, k_{\mathrm{m1}}, k_{\mathrm{m2}}, \alpha_{\mathrm{t}}$, and α_{m} may slightly vary with the difference of types, material, and technology of propellers. On the basis of the propeller data from T-MOTOR website [19], general parameters for carbon fiber propellers are given by:

$$k_{\mathrm{t0}} = 0.3462, k_{\mathrm{m0}} = 0.087, k_{\mathrm{m1}} = 0.01$$
$$k_{\mathrm{m2}} = 0.9, \alpha_{\mathrm{t}} = 0.9, \alpha_{\mathrm{m}} = 0.99. \tag{11}$$

2) Propeller Efficiency Objective Function: Similar to the lift–drag ratio for airfoils, a widely used efficiency index to describe the efficiency of propellers is $\eta_{\mathrm{T/M}}$, which is defined as the ratio between the thrust coefficient C_{T} and the torque coefficient C_{M} as:

$$\eta_{\mathrm{T/M}} \triangleq \frac{C_{\mathrm{T}}}{C_{\mathrm{M}}} = \frac{k_{\mathrm{t0}}\varphi_{\mathrm{p}}}{k_{\mathrm{m0}}B_{\mathrm{p}}^{(\alpha_{\mathrm{m}}-\alpha_{\mathrm{t}})}\left(k_{\mathrm{m1}}+k_{\mathrm{m2}}\varphi_{\mathrm{p}}^2\right)} \tag{12}$$

where $\eta_{\mathrm{T/M}}$ depends only on the aerodynamic design of the blade shape, which is convenient for manufacturers for improving the aerodynamic efficiency. Moreover, a higher $\eta_{\mathrm{T/M}}$ means a smaller torque for generating the same thrust. Because $\eta_{\mathrm{T/M}}$ is adopted by most manufacturers, this paper will use $\eta_{\mathrm{T/M}}$ as the propeller efficiency objective function to obtain the optimal φ_{pOpt} and B_{pOpt}.

3) Propeller Weight Objective Function: By analyzing the propeller products on the market, the propeller weight G_{p} can be described by a statistical model that depends on diameter D_{p} and the blade number B_{p} as:

$$G_{\mathrm{p}} = f_{G_{\mathrm{p}}}\left(B_{\mathrm{p}}, D_{\mathrm{p}}\right) \tag{13}$$

where $f_{G_{\mathrm{p}}}(\cdot)$ is an increasing function of D_{p} and B_{p}. Therefore, the minimum propeller weight, G_{p}, requires that both D_{p} and B_{p} should be chosen as small as possible, which is described as:

$$\min G_{\mathrm{p}} \Rightarrow \min B_{\mathrm{p}}, \min D_{\mathrm{p}}. \tag{14}$$

B. Motor Modeling

1) Motor Circuit Model: The equivalent circuit of a BLDC motor has been depicted in Fig. 3, where U_{m} (unit: V) is the motor equivalent voltage, and I_{m} (unit: A) is the motor equivalent current. According to [6] and [20], U_{m} and I_{m} can be obtained using the following:

$$\begin{cases} I_{\mathrm{m}} = \frac{\pi M K_{\mathrm{V}} U_{\mathrm{m0}}}{30(U_{\mathrm{m0}}-I_{\mathrm{m0}}R_{\mathrm{m}})} + I_{\mathrm{m0}} \\ U_{\mathrm{m}} = I_{\mathrm{m}}R_{\mathrm{m}} + \frac{U_{\mathrm{m0}}-I_{\mathrm{m0}}R_{\mathrm{m}}}{K_{\mathrm{V}}U_{\mathrm{m0}}} N \end{cases} \tag{15}$$

where M (unit: N·m) is the output torque of the motor, which is equal to the propeller toque in (8); N is the motor rotating speed, which is equal to the propeller rotating speed in (8); and the no-load voltage U_{m0} (unit: V) is a constant value defined by manufacturers (usually $U_{\mathrm{m0}} = 10$ V). Note that the nominal motor resistance, R_{m0}, on the product description is usually not accurate enough, and the nominal ESC resistance, R_{e0}, is usually unavailable on the product description. In this case, it is recommended to obtain the motor equivalent resistance, R_{m}, according to the experimental data with full-throttle current I_{m}^* and speed N^*. By letting $R_{\mathrm{e}} = 0$ for the ESC model in (30), a correction expression is derived from (15) as $R_{\mathrm{m}} \triangleq R_{\mathrm{e0}} + R_{\mathrm{m0}} \approx \left(U_{\mathrm{b}} - N^*/K_{\mathrm{V}}\right)/I_{\mathrm{m}}^* \approx 2 \sim 3R_{\mathrm{m0}}$ according to our experimental results.

2) Motor Constraints: The actual operating voltage of the motor depends on battery voltage U_{b} instead of the motor's nominal maximum voltage (NMV), U_{mMax}. The motor NMV, U_{mMax}, defines the range of the battery voltage U_{b} in which the safety of the motor work can be ensured; this range is described as $U_{\mathrm{b}} \leq U_{\mathrm{mMax}}$. According to [6], to prevent the motor from burnout, the motor equivalent voltage, U_{m}, and current I_{m} satisfy

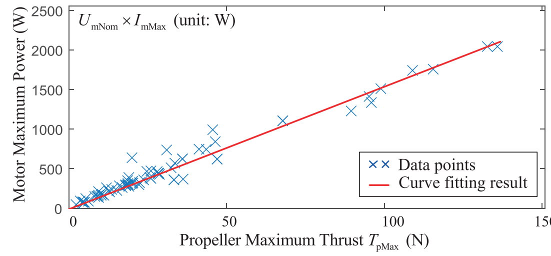

Fig. 4. Statistical relation between the motor's maximum power, $U_{\mathrm{mMax}} \cdot I_{\mathrm{mMax}}$, and propeller's maximum thrust, T_{pMax}, with the experimental data from [19].

the following:

$$\begin{aligned} U_{\mathrm{m}} &\leq U_{\mathrm{m}\sigma_{\max}} = U_{\mathrm{b}} \leq U_{\mathrm{mMax}} \\ I_{\mathrm{m}} &\leq I_{\mathrm{m}\sigma_{\max}} \leq I_{\mathrm{mMax}} \end{aligned} \tag{16}$$

where $U_{\mathrm{m}\sigma_{\max}}$ and $I_{\mathrm{m}\sigma_{\max}}$ are the motor voltage and current under the full-throttle mode ($\sigma_{\max} = 1$).

By letting the motor work under the maximum limit condition as the following:

$$U_{\mathrm{m}} = U_{\mathrm{mMax}}, I_{\mathrm{m}} = I_{\mathrm{mMax}}. \tag{17}$$

the maximum rotating speed, $N_{\max}$ (unit: RPM), and the maximum torque, $M_{\max}$ (unit: N·m), of the motor can be obtained by combining the motor model in (15) as follows:

$$\begin{cases} N_{\max} = f_{N_{\max}}\left(\Theta_{\mathrm{m}}\right) \triangleq \frac{(U_{\mathrm{mMax}}-R_{\mathrm{m}}I_{\mathrm{mMax}})K_{\mathrm{V}}U_{\mathrm{m0}}}{(U_{\mathrm{m0}}-I_{\mathrm{m0}}R_{\mathrm{m}})} \\ M_{\max} = f_{M_{\max}}\left(\Theta_{\mathrm{m}}\right) \triangleq \frac{30(I_{\mathrm{mMax}}-I_{\mathrm{m0}})(U_{\mathrm{m0}}-I_{\mathrm{m0}}R_{\mathrm{m}})}{\pi K_{\mathrm{V}}U_{\mathrm{m0}}} \end{cases}. \tag{18}$$

Moreover, according to the propeller model in (8), the theoretical maximum thrust, T_{pMax} (unit: N), can be obtained as:

$$T_{\mathrm{pMax}} = \frac{C_{\mathrm{T}}}{C_{\mathrm{M}}} \frac{M_{\max}^{4/5}\rho^{1/5}C_{\mathrm{M}}^{1/5}N_{\max}^{2/5}}{60^{2/5}}. \tag{19}$$

To satisfy the maneuverability requirement, the theoretical maximum thrust range, $[0, T_{\mathrm{pMax}}]$, of the propulsion system should cover the required thrust range $[0, T_{\max}]$, which means the following constraint should also be satisfied:

$$T_{\mathrm{pMax}} \geq T_{\max}. \tag{20}$$

According to the statistical results in Fig. 4, there is an equality constraint between the maximum input power, $U_{\mathrm{mMax}}I_{\mathrm{mMax}}$, and the theoretical maximum thrust, T_{pMax}, for motor products as:

$$\frac{T_{\mathrm{pMax}}}{U_{\mathrm{mMax}} \cdot I_{\mathrm{mMax}}} \approx G_{\mathrm{WConst}} \tag{21}$$

where G_{WConst} (unit: N/W) is a constant coefficient that reflects the technological process and product quality of products. According to the curve fitting result in Fig. 4, the coefficient for the tested motors is $G_{\mathrm{WConst}} \approx 0.0624$.

3) Motor Efficiency Objective Function: The motor power efficiency η_{m} is defined as:

$$\eta_{\mathrm{m}} \triangleq \frac{P_{\mathrm{p}}}{P_{\mathrm{m}}} = \frac{M\frac{2\pi N}{60}}{U_{\mathrm{m}} \cdot I_{\mathrm{m}}}. \tag{22}$$

According to (15), propeller torque M and rotating speed N can be described by $U_{\rm m}$ and $I_{\rm m}$, which yields:

$$\eta_{\rm m} = \left(1 - \frac{I_{\rm m}}{U_{\rm m}} R_{\rm m}\right)\left(1 - \frac{1}{I_{\rm m}} I_{\rm m0}\right). \tag{23}$$

It can be observed from (23) that motor efficiency $\eta_{\rm m}$ has negative correlations with $R_{\rm m}$ and $I_{\rm m0}$. Therefore, $R_{\rm m}$ and $I_{\rm m0}$ should be chosen as small as possible for the maximum motor efficiency, which is described as

$$\max \eta_{\rm m} \Rightarrow \min R_{\rm m}, \min I_{\rm m0}. \tag{24}$$

4) Weight Optimization Objective Function: By analyzing the motor products on the market, the motor weight, $G_{\rm m}$, can be described by a statistical model depending on the motor's NMV, $U_{\rm mMax}$, the nominal maximum current (NMC), $I_{\rm mMax}$, and the KV (motor velocity constant, measured in RPM per voltage) value, i.e., $K_{\rm V}$ as:

$$G_{\rm m} = f_{G_{\rm m}}\left(U_{\rm mMax}, I_{\rm mMax}, K_{\rm V}\right). \tag{25}$$

According to (19) and (21), $I_{\rm mMax}$ and $K_{\rm V}$ can be described by $U_{\rm mMax}$ and $T_{\rm pMax}$. Therefore, (25) can be rewritten into the following form:

$$G_{\rm m} = f'_{G_{\rm m}}\left(U_{\rm mMax}, T_{\rm pMax}\right). \tag{26}$$

Thus, by combining the constraints in (16) and (20), the motor-weight-optimization problem can be written as:

$$\min_{U_{\rm mMax}, T_{\rm pMax}} f'_{G_{\rm m}}\left(U_{\rm mMax}, T_{\rm pMax}\right)$$
$$\text{s.t. } U_{\rm b} \le U_{\rm mMax}, T_{\max} \le T_{\rm pMax}. \tag{27}$$

In practice, the motor weight, $G_{\rm m}$, has a positive correlation with $U_{\rm mMax}$ and $T_{\rm pMax}$. Therefore, the minimum motor weight, $G_{\rm m}$, requires that $U_{\rm mMax}$ and $T_{\rm pMax}$ should both be chosen as small as possible, which is described as:

$$\min G_{\rm m} \Rightarrow \min U_{\rm mMax}, \min T_{\rm pMax}. \tag{28}$$

Thus, solving (27) gives:

$$U_{\rm b} = U_{\rm mMax}, T_{\max} = T_{\rm pMax} \tag{29}$$

where $U_{\rm mMax}$ should be chosen as small as possible.

C. ESC Modeling

1) ESC Circuit Model: After receiving the throttle signal $\sigma \in [0, 1]$ from the flight controller, the ESC converts the direct-current power of the battery to the voltage modulated using pulsewidth modulation, $\sigma U_{\rm e}$, for the BLDC motor without speed feedback. Then, the rotating speed of the motor is determined both by the motor and propeller models. Because the propeller model is nonlinear, the motor speed is not in proportion to the input throttle signal. According to the ESC equivalent circuit in Fig. 3, the ESC current, $I_{\rm e}$ (unit: A), and the ESC voltage, $U_{\rm e}$ (unit: V), are given by:

$$\sigma U_{\rm e} = U_{\rm m} + I_{\rm m} R_{\rm e}$$
$$I_{\rm e} = \sigma I_{\rm m}. \tag{30}$$

2) ESC Efficiency Objective Function: According to (30), the power efficiency of the ESC can be obtained as:

$$\eta_{\rm e} \triangleq \frac{U_{\rm m} I_{\rm m}}{U_{\rm e} I_{\rm e}} = \frac{1}{1 + \frac{I_{\rm m}}{U_{\rm m}} R_{\rm e}} \tag{31}$$

which shows that the efficiency of the ESC, $\eta_{\rm e}$, increases as the resistance $R_{\rm e}$ decreases, which yields:

$$\max \eta_{\rm e} \Rightarrow \min R_{\rm e}. \tag{32}$$

3) ESC Weight Objective Function: By analyzing the ESC products on the market, the ESC weight, $G_{\rm e}$, can be described by a statistical model depending on the ESC's NMV, $U_{\rm eMax}$, and on the ESC's NMC, $I_{\rm eMax}$, as:

$$G_{\rm e} = f_{G_{\rm e}}\left(U_{\rm eMax}, I_{\rm eMax}\right) \tag{33}$$

where $f_{G_{\rm e}}(\cdot)$ is in positive correlation with $U_{\rm eMax}$ and $I_{\rm eMax}$. Therefore, to minimize $G_{\rm e}$, the ESC parameters $U_{\rm eMax}, I_{\rm eMax}$ should be chosen as small as possible, which is described as:

$$\min G_{\rm e} \Rightarrow \min U_{\rm eMax}, \min I_{\rm eMax}. \tag{34}$$

D. Battery Modeling

1) Battery Circuit Model: The battery is used to provide energy to drive the motor using ESC. The most commonly used type battery is the LiPo battery because of its superior performance and low price. According to Fig. 3, the battery model is given by:

$$U_{\rm b} = U_{\rm e} + I_{\rm b} R_{\rm b} \tag{35}$$

where $U_{\rm b}$ (unit: V) is the nominal battery voltage, and $I_{\rm b}$ (unit: A) is the output current.

Assuming that the number of the propulsion unit on a multicopter is $n_{\rm p}$, the battery current is given by:

$$I_{\rm b} = n_{\rm p} I_{\rm e} + I_{\rm other} \tag{36}$$

where $I_{\rm other}$ (unit: A) is the current from other devices on the multicopter, such as flight controller and camera. Usually, according to [3], it can be assumed that $I_{\rm other} \approx 0.5$ A if there is only a flight controller on the multicopter.

According to [6], the battery discharge time, $t_{\rm discharge}$ (unit: min), is determined by the capacity of the battery, $C_{\rm b}$, and the discharge current, $I_{\rm b}$ in the following:

$$t_{\rm discharge} \approx \frac{0.85 C_{\rm b}}{I_{\rm b}} \cdot \frac{60}{1000} \tag{37}$$

where the coefficient 0.85 denotes a 15% remaining capacity to avoid overdischarge. Note that the endurance computation equations in (35)–(37) are simplified to reduce the computation time. They can be replaced by more accurate and nonlinear methods as presented in [21] and [22] to increase the precision of the battery-optimization result.

2) Battery Constraints: The maximum discharge rate (MDR), $K_{\rm b}$, (unit: mA/mAh or marked with symbol "C") of the battery is defined as:

$$K_{\rm b} = \frac{1000 I_{\rm bMax}}{C_{\rm b}} \tag{38}$$

where I_{bMax} (unit: A) is the maximum discharge current that the battery can withstand. Since the battery should be able to work safely under the full-throttle mode of the motor, the maximum discharge current I_{bMax} should satisfy the following:

$$I_{\text{bMax}} \geq n_{\text{p}} I_{\text{e}\sigma_{\max}} + I_{\text{other}} = n_{\text{p}} I_{\text{mMax}} + I_{\text{other}} \tag{39}$$

which yields:

$$K_{\text{b}} \geq \frac{1000\left(n_{\text{p}} I_{\text{mMax}} + I_{\text{other}}\right)}{C_{\text{b}}}. \tag{40}$$

3) Battery Efficiency Objective Function: According to (35) and (36), battery efficiency, η_{b}, can be written as:

$$\eta_{\text{b}} \triangleq \frac{n_{\text{p}} U_{\text{e}} I_{\text{e}}}{U_{\text{b}} I_{\text{b}}} = \left(1 - \frac{I_{\text{b}}}{U_{\text{b}}} R_{\text{b}}\right)\left(1 - \frac{I_{\text{other}}}{I_{\text{b}}}\right) \tag{41}$$

which shows that the battery efficiency, η_{b}, increases as the resistance R_{b} decreases, which is described as:

$$\max \eta_{\text{b}} \Rightarrow \min R_{\text{b}}. \tag{42}$$

4) Battery Weight Objective Function: According to the definition of battery power density, ρ_{b} (unit: Wh/kg), the battery weight can be described as:

$$G_{\text{b}} = \frac{C_{\text{b}} U_{\text{b}}}{1000 g \rho_{\text{b}}} \tag{43}$$

where $g = 9.8$ m/s^2 is the acceleration due to gravity. Limited by battery technology, the power density, ρ_{b}, for a specific battery type is statistically close to a constant value. For example, $\rho_{\text{b}} \approx 140$ Wh/kg for LiPo batteries. Moreover, according to the statistical results, the battery weight is positively correlated with K_{b}. Therefore, to minimize G_{b}, the battery parameters C_{b}, U_{b}, and K_{b} should be chosen as small as possible, which is described as:

$$\min G_{\text{b}} \Rightarrow \min U_{\text{b}}, \min K_{\text{b}}, \min C_{\text{b}}. \tag{44}$$

IV. Design Optimization

A. Step 1: Propeller Efficiency Optimization

1) Optimal Blade Number B_{pOpt}: According to (12), if only blade number, B_{p}, is considered, the propeller thrust efficiency, $\eta_{\text{T/M}}$, can be simplified into the following form:

$$\eta_{\text{T/M}} \propto \frac{1}{B_{\text{p}}^{(\alpha_{\text{m}} - \alpha_{\text{t}})}} \tag{45}$$

where the symbol "$\propto$" means "in proportion to." (45) indicates that $\eta_{\text{T/M}}$ monotonically decreases as B_{p} increases because $\alpha_{\text{m}} > \alpha_{\text{t}}$ according to (10). Therefore, to maximize $\eta_{\text{T/M}}$, blade number, B_{p}, should be chosen as small as possible. Moreover, according to the weight-optimization principle in (14), the blade number should also be chosen as small as possible to minimize the propeller weight, G_{p}. Considering that the blade number should satisfy the constraint $B_{\text{p}} \geq 2$, the optimal blade number, B_{pOpt}, should be chosen as:

$$B_{\text{pOpt}} = 2. \tag{46}$$

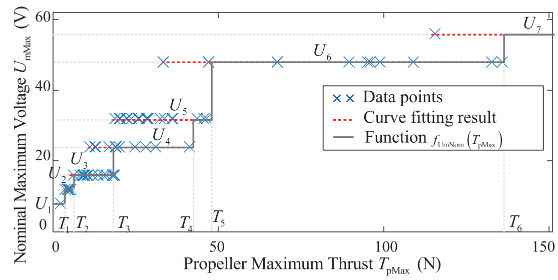

Fig. 5. Statistical relation between the motor's NMV, U_{mMax}, and the propeller's maximum thrust, T_{pMax}, with the experimental data from [19].

2) Optimal Pitch Angle, φ_{pOpt}: According to (12), if only the pitch angle φ_{p} is considered, $\eta_{\text{T/M}}$ can be simplified into the form as:

$$\eta_{\text{T/M}} \propto \frac{1}{k_{\text{m1}} \varphi_{\text{p}}^{-1} + k_{\text{m2}} \varphi_{\text{p}}}. \tag{47}$$

It is easy to obtain from (47) that $\eta_{\text{T/M}}$ first increases then decreases as φ_{p} increases. Therefore, the optimal pitch angle, φ_{pOpt}, should be able to maximize $\eta_{\text{T/M}}$, which yields:

$$\varphi_{\text{pOpt}} = \sqrt{\frac{k_{\text{m1}}}{k_{\text{m2}}}}. \tag{48}$$

Noteworthy, if it happens in some cases that there are few propeller products in the database with pitch angles close to the obtained φ_{pOpt}, then the mean pitch angle, $\overline{\varphi}_{\text{p}}$, can be chosen as the optimal pitch angle, $\varphi_{\text{pOpt}} = \overline{\varphi}_{\text{p}}$, to ensure that the method can find a proper propeller product.

B. Step 2: Motor-Weight Optimization

1) Optimal Value of the Motor's NMV, U_{mMaxOpt}: According to the statistical results in Fig. 5, the relation between U_{mMax} and T_{pMax} can be described as the following piecewise function:

$$f_{U_{\text{mMax}}}(T_{\text{pMax}}) \triangleq \begin{cases} U_1, & 0 < T_{\text{pMax}} \leq T_1 \\ U_2, & T_1 < T_{\text{pMax}} \leq T_2 \\ \cdots & \cdots \end{cases} \tag{49}$$

where $f_{U_{\text{mMax}}}(\cdot)$ is obtained with the principle of minimizing U_{mMax} for a given T_{pMax}. The procedures are presented as follows.

1) For each motor with nominal voltage, U_{mMax}, the maximum thrust, T_{pMax}, can be found on the product website or estimated by the methods in [6]. Then, a series of data points is obtained as marked with "$\times$" in Fig. 5.
2) The curve fitting methods are applied to each discrete voltage $(U_1, U_2, \ldots)$, and then several fitting results are obtained as represented by the dotted lines in Fig. 5.
3) When multiple voltages are available for a T_{pMax}, the minimum one should be chosen to minimize the motor weight. Then, it is easy to determine the thrust range $(T_i, T_{i+1}]$ for each voltage U_i. Finally, a piecewise function for $f_{U_{\text{mMax}}}(T_{\text{pMax}})$ can be obtained as marked with the solid lines in Fig. 5.

Thus, by substituting $T_{\text{pMax}} = T_{\text{max}}$ into (49), the optimal value of the motor's NMV, U_{mMaxOpt}, can be obtained as:

$$U_{\text{mMaxOpt}} = f_{U_{\text{mMax}}}(T_{\text{max}}). \tag{50}$$

2) Optimal NMC, $I_{mMaxOpt}$: According to (21), there is a function relation between T_{pMax}, I_{mMax}, and U_{mMax}. Therefore, by substituting $U_{\text{mMax}} = U_{\text{mMaxOpt}}$ and $T_{\text{pMax}} = T_{\text{max}}$ into (21), the optimal value of the motor's NMC, I_{mMaxOpt}, can be obtained as:

$$I_{\text{mMaxOpt}} = \frac{T_{\text{max}}}{G_{\text{WConst}} U_{\text{mMaxOpt}}}. \tag{51}$$

3) Optimal KV Value, K_{VOpt}: Because the motor's no-load current, I_{m0}, and resistance R_{m} are very small in practice, it is reasonable to assume that:

$$I_{\text{m0}} \approx 0 \text{ and } R_{\text{m}} \approx 0. \tag{52}$$

Substituting (52) into (18) gives:

$$M_{\max} \approx \frac{30 I_{\text{mMax}}}{\pi K_{\text{V}}}$$
$$N_{\max} \approx U_{\text{mMax}} K_{\text{V}}. \tag{53}$$

Thus, (19) can be simplified into the following form:

$$T_{\text{pMax}} \approx k_{\text{tm}} \cdot \left(\frac{I_{\text{mMax}}^2 U_{\text{mMax}}}{K_{\text{V}}} \right)^{2/5} \tag{54}$$

where k_{tm} is defined as:

$$k_{\text{tm}} = f_{k_{\text{tm}}}(B_{\text{p}}, \varphi_{\text{p}}, h_{\text{hover}}) \triangleq \sqrt[5]{k_{\text{c}} \frac{255 \rho C_{\text{T}}^5}{\pi^4 C_{\text{M}}^4}} \tag{55}$$

where $\rho = f_\rho(h_{\text{hover}})$, $C_{\text{T}} = f_{C_{\text{T}}}(B_{\text{p}}, D_{\text{p}}, \varphi_{\text{p}})$, and $C_{\text{M}} = f_{C_{\text{M}}}(B_{\text{p}}, D_{\text{p}}, \varphi_{\text{p}})$ are defined in (9) and (10), and k_{c} is a constant correction coefficient to compensate for the neglected factors including I_{m0} and R_{m}. According to the statistical analysis, the correction coefficient can be set as $k_{\text{c}} \approx 0.82$.

Finally, by substituting $U_{\text{mMaxOpt}}, I_{\text{mMaxOpt}}, T_{\text{max}}, B_{\text{pOpt}}, \varphi_{\text{pOpt}}$, and h_{hover} into (54) and (55), the expression for the optimal KV value, K_{VOpt}, is obtained as:

$$K_{\text{VOpt}} = f_{k_{\text{tm}}}^{5/2}(B_{\text{pOpt}}, \varphi_{\text{pOpt}}, h_{\text{hover}}) \frac{I_{\text{mMaxOpt}}^2 U_{\text{mMaxOpt}}}{T_{\text{max}}^{5/2}}. \tag{56}$$

C. Step 3: Motor-Efficiency Optimization

1) Optimal Motor Resistance, R_{mOpt}, and No-Load Current, I_{m0Opt}: In practice, R_{m} and I_{m0} should satisfy the following constraint:

$$R_{\text{m}} > 0 \text{ and } I_{\text{m0}} > 0. \tag{57}$$

According to (24), both the optimal motor resistance, R_{mOpt}, and the optimal no-load current, I_{m0Opt}, should be selected as small as possible. Constrained by (57), the optimal motor resistance, R_{mOpt}, and the optimal no-load current, I_{m0Opt}, are marked with the following:

$$R_{\text{mOpt}} = 0 \text{ and } I_{\text{m0Opt}} = 0 \tag{58}$$

which denotes that the motor resistance and no-load current should be chosen as close to zero as possible.

D. Step 4: Optimal Motor Product

Although the optimal motor parameters $\Theta_{\text{mOpt}} \triangleq \{U_{\text{mMaxOpt}}, I_{\text{mMaxOpt}}, K_{\text{VOpt}}, R_{\text{mOpt}}, I_{\text{m0Opt}}\}$ have been obtained using the aforementioned procedures, yet it is difficult to determine a corresponding product from the database. For example, the values of U_{mMax} of motor products are usually given by discrete forms such as 20, 30, and 40 A, whereas the obtained solutions are usually given with continuous forms such as $I_{\text{mMaxOpt}} = 33.5$ A. To solve this problem, a method is proposed to determine the optimal motor product according to Θ_{mOpt}. For simplicity, the parameter set of the obtained motor product is represented by $\Theta_{\text{mOpt}}^* \triangleq \{U_{\text{mMaxOpt}}^*, I_{\text{mMaxOpt}}^*, K_{\text{VOpt}}^*, R_{\text{mOpt}}^*, I_{\text{m0Opt}}^*\}$.

There are two selection principles for the optimal motor product.

1) The product should be selected by comparing with every parameter of Θ_{mOpt} in a proper sequence. Using the statistical analysis of the motor products on the market, a comparison sequence is given by considering the influence of each parameter on the motor weight as:

$$U_{\text{mMaxOpt}}^*, K_{\text{VOpt}}^*, I_{\text{mMaxOpt}}^*, R_{\text{mOpt}}^*, I_{\text{m0Opt}}^*. \tag{59}$$

2) When comparing a parameter, for ensuring safety requirements, the product should be chosen equal to or close to the corresponding parameter in Θ_{mOpt}. For example, if $I_{\text{mMaxOpt}} = 33.5$ A and the available current options are 20, 30, and 40 A, then it should be chosen that $I_{\text{mMaxOpt}}^* = 40$ A for some safety margin. The safety constraints for the selection of motor products are given by:

$$U_{\text{mMaxOpt}}^* \geq U_{\text{mMaxOpt}}, I_{\text{mMaxOpt}}^* \geq I_{\text{mMaxOpt}}. \tag{60}$$

E. Step 5: Propeller-Weight Optimization

1) Optimal Diameter, D_{pOpt}: According to (20), the following constraint should be satisfied:

$$T_{\text{pMax}} = C_{\text{T}} \rho \left(\frac{N_{\text{max}}}{60} \right)^2 D_{\text{p}}^4 \geq T_{\text{max}} \tag{61}$$

which yields:

$$D_{\text{p}} \geq \sqrt[4]{\frac{60^2 T_{\text{max}}}{C_{\text{T}} \rho N_{\text{max}}^2}}. \tag{62}$$

According to the optimization objective in (14), the optimal diameter should be chosen as the minimum diameter under the constraint in (62). Therefore, the optimal diameter, D_{pOpt}, can be obtained by combining (9), (10), and (18) with parameters h_{hover}, D_{pOpt}, B_{pOpt}, and Θ_{mOpt}^* as:

$$D_{\text{pOpt}} = \sqrt[4]{\frac{60^2 T_{\text{max}}}{\rho C_{\text{T}} N_{\text{max}}^2}} = \sqrt[4]{\frac{60^2 T_{\text{pMax}}}{\rho C_{\text{T}} N_{\text{max}}^2}} = \sqrt[5]{\frac{M_{\max}}{\rho C_{\text{M}} \left(\frac{N_{\max}}{60}\right)^2}}$$
$$= \sqrt[5]{\frac{3600 f_{M_{\max}}\left(\Theta_{\text{mOpt}}^*\right)}{f_\rho(h_{\text{hover}}) f_{C_{\text{M}}}(B_{\text{pOpt}}, D_{\text{pOpt}}) f_{N_{\max}}^2\left(\Theta_{\text{mOpt}}^*\right)}}. \tag{63}$$

Because the propeller pitch, $H_{\rm p}$, is more convenient to select a propeller product, according to the definition of the pitch angle, $\varphi_{\rm p}$, in (3), the optimal propeller pitch, $H_{\rm pOpt}$, is given by:

$$H_{\rm pOpt} = \pi \cdot D_{\rm pOpt} \cdot \tan\varphi_{\rm pOpt}.$$

F. Step 6: Optimal Propeller Product

With the obtained parameters $\Theta_{\rm pOpt} = \{B_{\rm pOpt}, \varphi_{\rm p}, D_{\rm pOpt}\}$, the optimal propeller product can be determined by searching the propeller product database. The parameter set of the obtained optimal propeller product is represented by $\Theta^*_{\rm pOpt} = \{B^*_{\rm pOpt}, \varphi^*_{\rm p}, D^*_{\rm pOpt}\}$.

Similar to the selection principles in Section IV-D, the optimal propeller product should be determined from the propeller database by comparing the parameters in the sequence $B^*_{\rm pOpt}$, $\varphi^*_{\rm p}$, $D^*_{\rm pOpt}$, and each parameter in $\Theta^*_{\rm pOpt}$ should be chosen equal or close to the corresponding parameter in $\Theta_{\rm pOpt}$ while satisfying the following safety constraints:

$$B^*_{\rm pOpt} = B_{\rm pOpt},\ D^*_{\rm pOpt} \le D_{\rm pOpt}\,. \tag{64}$$

G. Step 7: ESC-Weight Optimization

1) Optimal ESC's NMV, $U_{\rm eMaxOpt}$ and Optimal ESC's NMC, $I_{\rm eMaxOpt}$:

Because the ESC and the motor are connected in series, their voltage and current should match with each other to ensure proper operations. For safety, the ESC's NMV, $U_{\rm eMax}$, and the ESC's NMC, $I_{\rm eMax}$, should be able to support the maximum voltage and current from the motor, which means:

$$U_{\rm eMax} \ge U_{\rm mMaxOpt},\ I_{\rm eMax} \ge I_{\rm mMaxOpt}\,. \tag{65}$$

By combining (34) and (65), the $U_{\rm eMaxOpt}$ and $I_{\rm eMaxOpt}$ are given by:

$$U_{\rm eMaxOpt} = U_{\rm mMaxOpt},\ I_{\rm eMaxOpt} = I_{\rm mMaxOpt}\,. \tag{66}$$

H. Step 8: ESC-Efficiency Optimization

1) Optimal ESC Resistance, $R_{\rm eOpt}$:

According to (32), the optimal ESC resistance, $R_{\rm eOpt}$, should be chosen as small as possible for the maximum ESC efficiency, $\eta_{\rm e}$. Because $R_{\rm e} > 0$ in practice, the optimal ESC resistance, $R_{\rm eOpt}$, is marked with the following:

$$R_{\rm eOpt} = 0 \tag{67}$$

which denotes that the ESC resistance should be chosen as close to zero as possible.

I. Step 9: Optimal ESC Product

With the obtained parameters $U_{\rm eMaxOpt}, I_{\rm eMaxOpt}$, and $R_{\rm eOpt}$, the optimal ESC product can be determined by searching the ESC product database. The parameter set of the obtained optimal ESC product is represented by $\Theta^*_{\rm eOpt} = \{U^*_{\rm eMaxOpt}, I^*_{\rm eMaxOpt}, R^*_{\rm eOpt}\}$.

Similar to the selection principles in Section IV-D, the optimal ESC product should be determined from the ESC database by comparing the parameters in the sequence $U^*_{\rm eMaxOpt}, I^*_{\rm eMaxOpt}, R^*_{\rm eOpt}$, and each parameter in $\Theta^*_{\rm eOpt}$ should be chosen equal or close to the corresponding parameter in $\Theta_{\rm eOpt}$ while satisfying the following constraints:

$$U^*_{\rm eMaxOpt} \ge U_{\rm eMaxOpt},\ I^*_{\rm eMaxOpt} \ge I_{\rm eMaxOpt}\,. \tag{68}$$

J. Step 10: Battery-Efficiency Optimization

1) Optimal Battery Resistance, $R_{\rm bOpt}$:

According to (42), $R_{\rm b}$ should be chosen as small as possible for the maximum battery efficiency, $\eta_{\rm b}$. Considering that $R_{\rm b} > 0$ in practice, the optimal battery resistance, $R_{\rm bOpt}$ is marked with the following:

$$R_{\rm bOpt} = 0 \tag{69}$$

which denotes that the battery resistance should be chosen as close to zero as possible.

K. Step 11: Battery-Weight Optimization

1) Optimal Battery Nominal Voltage, $U_{\rm bOpt}$:

As analyzed in Section IV-B, the actual working voltage of the motor is determined by battery voltage, and the constraint in (29) should be satisfied to make sure the motor has the minimum weight. Therefore, after the optimal value of the motor's NMV, $U_{\rm mMaxOpt}$, is determined, the optimal battery voltage, $U_{\rm bOpt}$, is also determined as:

$$U_{\rm bOpt} = U_{\rm mMaxOpt}. \tag{70}$$

2) Optimal Battery Capacity, $C_{\rm bOpt}$:

After the aforementioned procedures, the propeller, motor, ESC, and battery of the propulsion system have the maximum efficiency, which means the battery has the minimum current, $I_{\rm b0}$, under the hovering mode with propeller thrust, $T_{\rm hover}$. According to [6], with knowing the parameters $\Theta^*_{\rm pOpt}, \Theta^*_{\rm mOpt}, \Theta^*_{\rm eOpt}, U_{\rm bOpt}, T_{\rm hover}, n_{\rm p}$, and $h_{\rm hover}$, the battery current, $I_{\rm b0}$, can be estimated using the propulsion-system equivalent circuit in Fig. 3. By substituting the desired propeller thrust, $T_{\rm hover}$, into the propeller, motor, ESC, and battery models in (8), (15), (30), and (35) successively, the battery discharge current, $I_{\rm b0}$, can be obtained. Then, the optimal battery capacity, $C_{\rm bOpt}$, can be obtained by substituting $t_{\rm discharge} = t_{\rm hover}$ into (37), which yields:

$$C_{\rm bOpt} = \frac{t_{\rm hover} I_{\rm b0}}{0.85}\frac{1000}{60}. \tag{71}$$

3) Optimal MDR, $K_{\rm bOpt}$:

The MDR of the battery, $K_{\rm b}$, should be chosen as small as possible within the constraint in (40). Therefore, by substituting the obtained $U_{\rm mMaxOpt}$ and $C_{\rm bOpt}$ into (40), the optimal battery MDR, $\mathrm{K}_{\rm bOpt}$, is given by:

$$K_{\rm b} = \frac{1000\,(n_{\rm p} I_{\rm mMaxOpt} + I_{\rm other})}{C_{\rm bOpt}}. \tag{72}$$

L. Step 12: Optimal Battery Product

With the obtained parameters $\Theta_{\rm bOpt} = \{U_{\rm bOpt}, K_{\rm bOpt}, C_{\rm bOpt}, R_{\rm bOpt}\}$, the optimal battery product can be determined by searching the battery product database. The parameter set of the obtained optimal battery product is represented by $\Theta^*_{\rm bOpt} = \{U^*_{\rm bOpt}, K^*_{\rm bOpt}, C^*_{\rm bOpt}, R^*_{\rm bOpt}\}$.

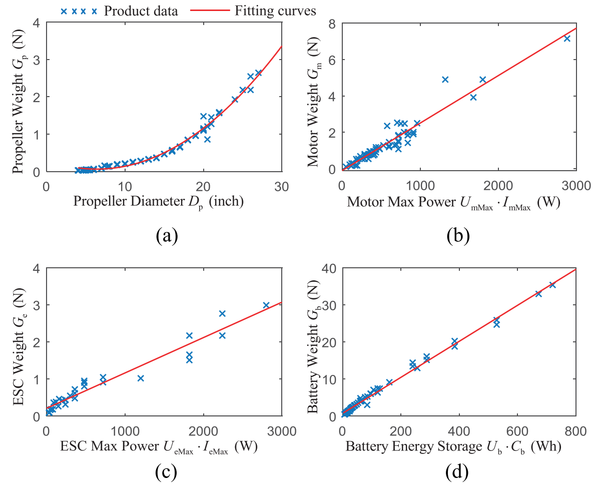

Fig. 6. Weight statistical results for the propulsion-system components. (a) Propeller weight statistical result with data from APC E-series propellers. (b) Motor weight statistical result with data from T-MOTOR BLDC motors. (c) ESC weight statistical result with data from Hobbywing UAV ESCs. (d) Battery weight statistical result with data from ACE LiPo batteries.

Similar to the selection principles in Section IV-D, the optimal battery product should be determined from the battery database by comparing the parameters in the sequence $U^*_{\rm bOpt}, K^*_{\rm bOpt}, C^*_{\rm bOpt}, R^*_{\rm bOpt}$, and each parameter in $\Theta^*_{\rm bOpt}$ should be chosen equal or close to the corresponding parameter in $\Theta_{\rm bOpt}$ while satisfying the following constraints:

$$U^*_{\rm bOpt} = U_{\rm bOpt},\ K^*_{\rm bOpt} \geq K_{\rm bOpt}\,. \tag{73}$$

Noteworthy, the battery voltage, $U^*_{\rm bOpt}$, should satisfy the constraint $U^*_{\rm bOpt} = U_{\rm bOpt} = U_{\rm mMaxOpt}$ to ensure that the motor can work under the desired voltage.

In practice, designers have to build a battery pack to satisfy the aforementioned design requirements by connecting small battery cells in series or parallel. On the one hand, according to [3, p. 46], by combining battery cells in series, a higher voltage can be obtained, with capacity unchanged. On the other hand, by combining battery cells in parallel, larger capacity and discharge current can be obtained, with voltage unchanged.

V. Experiments and Verification

In our previous research [6], a measuring apparatus was introduced to measure the parameters of multicopter propulsion systems. Meanwhile, the experimental data published on the product websites of manufacturers are also used for verifying the proposed method.

A. Statistical Model Verification

Comprehensive statistical analyses for the products of propellers, motors, ESCs, and batteries on the market are performed to verify the weight statistical functions $f_{G_{\rm p}}(\cdot), f_{G_{\rm m}}(\cdot)$, $f_{G_{\rm e}}(\cdot)$, and $f_{G_{\rm b}}(\cdot)$ in (13), (25), (33), and (43), respectively. Some typical results are presented in Fig. 6, where the products come from the four most well-known manufacturers (APC, T-MOTOR, Hobbywing, Gens ACE). Fig. 6 shows the relation between the weight and the parameters of each component.

TABLE II
Diameter/Pitch Ratio of Carbon Fiber Propellers From the Website [19]

Propeller	P12x4	P14x4.8	P15x5	P16x5.4
$D_{\rm p}/H_{\rm p}$	3	2.92	3	2.963
Propeller	P18x6.1	···	G26x8.5	G28x9.2
$D_{\rm p}/H_{\rm p}$	2.951	···	3.06	3.04
Propeller	G30x10	G32x11	G34x11.5	G40x13.1
$D_{\rm p}/H_{\rm p}$	3	2.91	2.96	3.05

TABLE III
Full-Throttle Test Data of U11 KV90 With 12S LiPo Battery (48 V)

Prop.	Current (A)	Power (W)	Thrust (N)	RPM	Torque (N·m)	Tempe. (°C)
27x8.8CF	24.6	1180.8	81.4	3782	2.623	58.5
28x9.2CF	28.3	1358.4	91.3	3696	3.068	66.5
29x9.5CF	31.9	1531.2	98.8	3602	3.41	78.5
30x10.5CF	36.3	1742.4	106.8	3503	3.846	HOT!

The statistical results are consistent with the analysis results in Section IV.

B. Optimization Method Verification

According to (48), the optimal pitch angle for T-MOTOR series propellers [parameters are listed in (11)] is obtained as $\varphi_{\rm pOpt} = 0.1054$ rad. For comparative validation, a series of multicopter propellers from the website [19] is listed in Table II. It can be observed from Table II that the statistical diameter/pitch ratio result is $D_{\rm p}/H_{\rm p} \approx 3$ ($\varphi_{\rm p} \approx 0.1065$ rad), which is in good agreement with the theoretical result, $\varphi_{\rm pOpt} = 0.1054$ rad. The comparison between the calculation value and the statistical value shows that the pitch-angle-optimization method is effective to find the optimal pitch angle adopted by manufacturers. Meanwhile, the propeller test data from the UIUC website [18] also show that the obtained pitch angle, $\varphi_{\rm pOpt}$, can guarantee high efficiency in increasing the thrust and decreasing the torque.

The motor T-MOTOR U11 KV90 is adopted as an example to verify the proposed optimization method. The calibrated motor parameters of U11 KV90 are listed as follows:

$$\begin{aligned} &K_{\rm V} = 90\,\text{RPM/V},\ U_{\rm mMax} = 48\,\text{V},\ I_{\rm mMax} = 36\,\text{A} \\ &U_{\rm m0} = 10\,\text{V},\ I_{\rm m0} = 0.7\,\text{A},\ R_{\rm m} = 0.3\,\Omega \end{aligned} \tag{74}$$

and the corresponding experiment results from [19] are listed in Table III.

From the perspective of theoretical calculation, the optimal diameter of motor U11 KV90 can be obtained by substituting the motor parameters in (74) into (63), where the obtained result is $D_{\rm pOpt} = 29.7$ in. Therefore, according to the constraint in (64), the propeller diameter should be chosen as $D^*_{\rm pOpt} = 29$ in because the motor will overheat if the propeller diameter is greater than $D_{\rm pOpt}$. It can be observed from the experimental results in the last two rows of Table III that the motor temperature becomes overheated when the propeller diameter changes from 29 to 30 in. Therefore, the optimal propeller diameter obtained from experiments should be 29 in, which agrees with the theoretical optimal solution, $D^*_{\rm pOpt} = 29$ in.

C. Design-Optimization Example

As an example, assume that the given design task is to select an optimal propulsion system for a multicopter ($n_{\rm p} = 4$) whose total weight, $G_{\rm total} = 196$ N (20 kg), and flight time is $t_{\rm hover} = 17$ min (flight altitude, $h_{\rm hover} = 50$ m). The component databases are composed of the ESC, BLDC motor, and carbon fiber propeller products from T-MOTOR website [19], and the LiPo battery products from GENS ACE website [23].

The key calculation results of the design procedures are listed as follows.

1) The thrust requirements for the propulsion system are obtained from (1) and (2) as $T_{\rm hover} = 49$ N and $T_{\rm max} = 98$ N, where $\gamma = 0.5$ is adopted here.
2) The optimal propeller efficiency parameters are obtained from (46) and (48) as $B_{\rm pOpt} = 2$ and $\varphi_{\rm pOpt} = 0.1065$ rad. Then, with statistical models in Figs. 4 and 5, the optimal motor parameters are obtained from (50), (51), and (56) as $U_{\rm mMaxOpt} = 48$ V, $I_{\rm mMaxOpt} = 34$ A, $K_{\rm VOpt} = 91$ RPM/V. Therefore, by searching products from the T-MOTOR website according to the principles in (59) and (60), the optimal motor is determined as U11 KV90.
3) With the blade parameters in (11), the optimal propeller diameter can be obtained from (63) as $D_{\rm pOpt} = 0.7468$ m, and the optimal propeller product is selected as 29 × 9.5 CF 2-blade. In the same way, the ESC parameters can also be obtained from (66) as $U_{\rm eMaxOpt} = 48$ V and $I_{\rm eMaxOpt} = 34$ A, and the optimal ESC product is selected as FLAME 60A HV.
4) For the battery, the optimal parameters are obtained from (70), (71), and (72) as $U_{\rm bOpt} = 48$ V, $C_{\rm bOpt} = 16000$ mAh, and $K_{\rm bOpt} = 10$ C. The optimal battery selected from ACE website is TATTU LiPo 6S 15C 16000 mAh × 2.

The obtained result has been verified by several multicopter designers. Experiments show that a quadcopter with the designed propulsion system can efficiently meet the desired design requirements. If a larger motor (such as T-MOTOR U13) is selected, then a smaller propeller has to be chosen for generating the same full-throttle thrust, $T_{\rm pMax}$, which reduces the motor efficiency because the optimal operating condition cannot be reached. As a result, the obtained propulsion system is heavier than the optimal result according to our experiments. If a smaller motor is selected, then a larger propeller has to be chosen, which results in exceeding the safety current of the motor. Therefore, the obtained propulsion system is optimal within the given database.

D. Method Application

The optimization algorithm proposed in this paper has been published as a subfunction in an online toolbox (URL: www.flyeval.com/recalc.html) to estimate the optimal propulsion system with the given design requirements. The program is fast enough to be finished within 30 ms using a web server with low configuration (single-core CPU and 1 GB of RAM). The feedback results from the users show that the optimization results are effective and practical for multicopter design.

VI. Conclusion

In this paper, the precise modeling methods for the propeller, ESC, motor, and battery are studied, respectively, to solve the optimization problem for the propulsion system of multicopters. Then, the key parameters of each component are estimated using mathematical derivations to make sure that the obtained propulsion system has the maximum efficiency. Experiments and feedback on the website demonstrate the effectiveness of the proposed method. The propulsion system is the most important part of a multicopter, and its design-optimization method will be conducive to the fast, optimal, and automatic design of the whole multicopter system or other types of aircraft systems. The theoretical analysis can be further used to directly maximize the endurance of all kinds of UAVs, which is interesting for future research.

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